Sec 2.2 Stability and equilibrium

ODE is autonomous if $\frac{dy}{dt} = f(y)$ dep. var. only, $\left(\Rightarrow \frac{dy}{f(y)} = dt\right)$

Popul. models $\frac{dP}{dt} = kP$, $\frac{dP}{dt} = kP(M-P)$

are autonomous

"Autonomous" $\Rightarrow$ only "height of P (or y)" affects change.

critical points of ODE are y-values such that

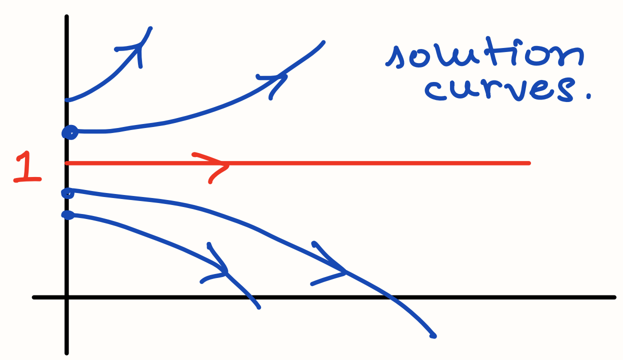
$f(y) = 0 \quad \left(\Rightarrow \frac{dy}{dt} = 0\right)$

Ex: $\frac{dy}{dt} = (y-1)$

$f(y) = y-1$;

crit pt @ $y = 1$

$$\begin{cases} y<1 \Rightarrow y'<0 \\ y=1 \Rightarrow y'=0 \\ y>1 \Rightarrow y'>0 \end{cases}$$

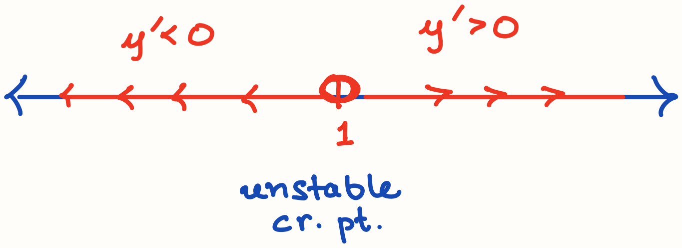


(separable ... if $y(0) = y_0$, then sols are $y(t) = 1 + (y_0 - 1)e^t$)

Def: If c is a crit. pt. then the solution $y(t) \equiv c$ is an equilibrium solution

Def: If solution curves are repelled away from crit. pt $y=c$ on both sides, then $y=c$ is an unstable crit. pt.

The above example / situation can be visually described by a <u>phase diagram</u>:



<u>Ex</u> $\frac{dy}{dt} = y(y-1)^2$

★ Can examine stable/unstable cr. pts. etc. without solving ODE.

cr. pts : $y = 0, 1$.

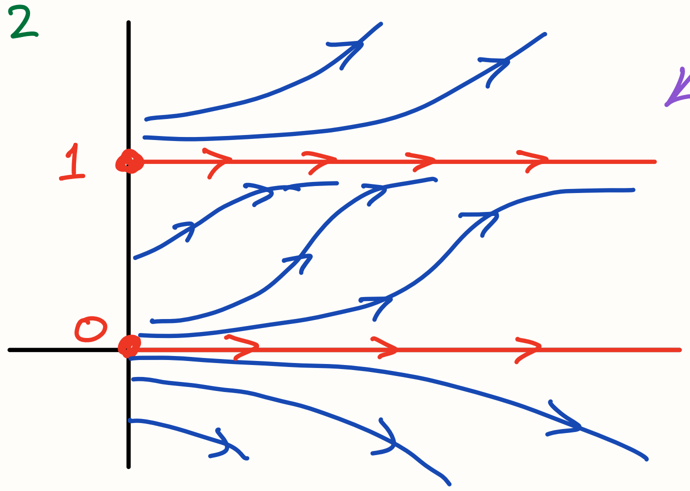
<u>equilibrium sols</u> $y(t) \equiv 0$, $y(t) \equiv 1$

- Check y' @ $y=-1$, $y=\frac{1}{2}$, $y=2$

$y=-1 \Longrightarrow y' = (-)(-)^2 = (-)$

$y=\frac{1}{2} \Longrightarrow y' = (+)(-)^2 = +$

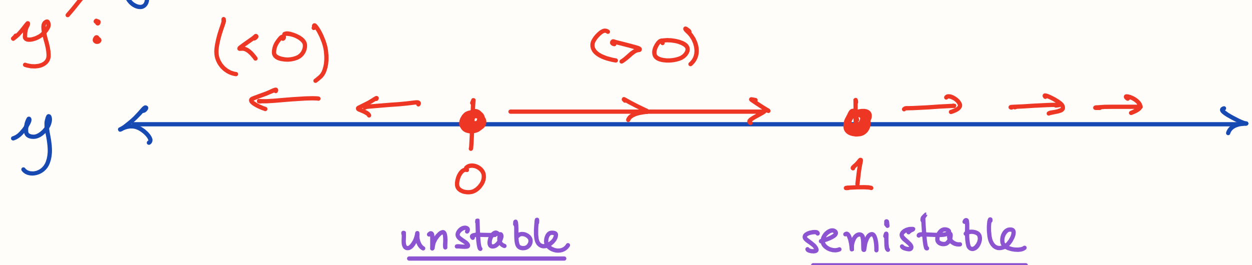
$y = 2$



$y=1$ is a <u>semistable</u> cr pt (one side attracted, one repelled)

$y=0$ is <u>unstable</u> cr. pt.

<u>phase diagram</u>



Ex $\dfrac{dy}{dt} = (y-1)(y+2)(y^2-9)$

critical points are $y = 1, -2, -3, 3$